Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 9 / 0 1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 24 June 2013 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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P 4 3 1 4 3 A 0 1 2 8

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PEARSON

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1. A hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{25} = 1, \qquad \text{where } a \text{ is a positive constant.}$$

The foci of H are at the points with coordinates (13, 0) and (–13, 0).

Find

(a) the value of the constant a,

(3)

(b) the equations of the directrices of H.

(3)

P 4 3 1 4 3 A 0 2 2 8

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Question 1 continued

Q1

(Total 6 marks)

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2. (a) Find

$$\int \frac{1}{\sqrt{(4x^2 + 9)}}\,\mathrm{d}x$$

(2)

(b) Use your answer to part (a) to find the exact value of

$$\int_{-3}^{3} \frac{1}{\sqrt{(4x^2 + 9)}}\,\mathrm{d}x$$

giving your answer in the form $k \ln(a + b\sqrt{5})$, where a and b are integers and k is a constant.

(3)

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Question 2 continued

P 4 3 1 4 3 A 0 5 2 8

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Question 2 continued

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Question 2 continued

Q2

(Total 5 marks)

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3. The curve with parametric equations

$$x = \cosh 2\theta, \quad y = 4 \sinh \theta, \quad 0 \leqslant \theta \leqslant 1$$

is rotated through 2π radians about the x-axis.

Show that the area of the surface generated is $\lambda(\cosh^3 \alpha - 1)$, where $\alpha = 1$ and λ is a constant to be found.

(7)

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Question 3 continued

Q3

(Total 7 marks)

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4.

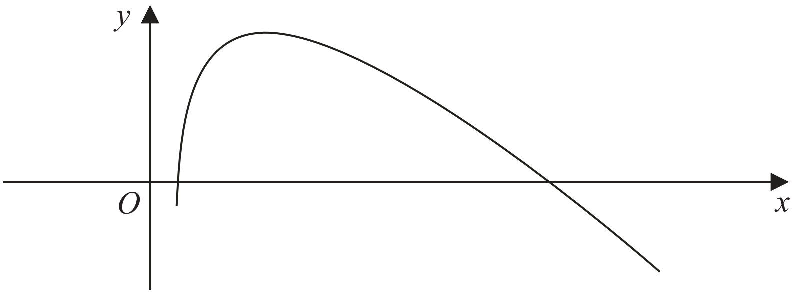

Figure 1

Figure 1 shows part of the curve with equation

$$y = 40 \text{ arcosh } x - 9x, \qquad x \geqslant 1$$

Use calculus to find the exact coordinates of the turning point of the curve, giving your answer in the form $\left(\frac{p}{q}, r\ln 3 + s\right)$, where p, q, r and s are integers.

(7)

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Question 4 continued

Q4

(Total 7 marks)

P 4 3 1 4 3 A 0 1 1 2 8

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5. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & a \\ 2 & b & c \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } a, b \text{ and } c \text{ are constants.}$$

(a) Given that **j** + **k** and **i** – **k** are two of the eigenvectors of **M**,

find

(i) the values of a, b and c,

(ii) the eigenvalues which correspond to the two given eigenvectors.

(8)

(b) The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & d \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1$$

Find

(i) the determinant of **P** in terms of d,

(ii) the matrix $\mathbf{P}^{-1}$ in terms of d.

(5)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 13 marks)

P 4 3 1 4 3 A 0 1 5 2 8

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6. Given that

$$I_n = \int_0^4 x^n \sqrt{(16 - x^2)}\,\mathrm{d}x, \quad n \geqslant 0,$$

(a) prove that, for $n \geqslant 2$,

$$(n+2)I_n = 16(n-1)I_{n-2}$$

(6)

(b) Hence, showing each step of your working, find the exact value of I_5

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

Q6

(Total 11 marks)

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7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad a > b > 0$$

The line l is a normal to E at a point $P(a\cos\theta, b\sin\theta)$, $\quad 0 < \theta < \frac{\pi}{2}$

(a) Using calculus, show that an equation for l is

$$ax\sin\theta - by\cos\theta = (a^2 - b^2)\sin\theta\cos\theta$$

(5)

The line l meets the x-axis at A and the y-axis at B.

(b) Show that the area of the triangle OAB, where O is the origin, may be written as $k\sin 2\theta$, giving the value of the constant k in terms of a and b.

(4)

(c) Find, in terms of a and b, the exact coordinates of the point P, for which the area of the triangle OAB is a maximum.

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 12 marks)

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8. The plane Π_1 has vector equation

$$\mathbf{r}.(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) = 5$$

(a) Find the perpendicular distance from the point (6, 2, 12) to the plane Π_1

(3)

The plane Π_2 has vector equation

$$\mathbf{r} = \lambda(2\mathbf{i} + \mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k}),$$ where λ and μ are scalar parameters.

(b) Find the acute angle between Π_1 and Π_2 giving your answer to the nearest degree.

(5)

(c) Find an equation of the line of intersection of the two planes in the form $\mathbf{r} \times \mathbf{a} = \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors.

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

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